Feasibility of Teaching Equivalent Simultaneous Linear Equations for Solving Quadratic Equations

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Abstract - *This paper examined the feasibility of teaching the method of equivalent simultaneous linear equations (ELSE) in Senior High School 1. The study tried to find out if there is a significant difference between the scores of students who used ELSE and those who used the conventional method. Also, if students who used ELSE can remember and applied it as compared to those who used the conventional methods. A sample of 72 students Akro Secondary Technical School, Eastern Region of Ghana took part in this feasibility study and the sample consisted of 42 boys and 30 girls. The reliability coefficients of the tests were determined by the Cronbach Alpha method and Wilcoxon Rank-Sum Test was also used to test if the performance between both groups were significantly different or not. From the results, the reliability coefficients for all tests were above 70 percent. The results of the performance of the experimental group were better than that of the control group in test IV to VI. This showed that students in the experimental group learnt and remember the ELSE and applied it more than their counterpart in the control group. In conclusion, the ELSE has the power that overcomes the inherent problems of the conventional methods of solving quadratic equations. Therefore, the ELSE holds a promise for mathematics education. As a result, a nationwide study is recommended to verify on a large scale the feasibility of the ELSE and subsequent inclusion in the core mathematics curriculum to help solved the problem of solving quadratic equation.*

***Keywords**: Feasibility, The Method of Equivalent Simultaneous Linear Equations, Cronbach Alpha Method, Wilcoxon Rank-Sum Test, Reliability Coefficients*

I. INTRODUCTION

The importance of teaching mathematics in schools can be generalized into four main categories. These are the utilitarian value, personal value, social and cultural values. In the pass mankind tried to develop language for easy communication and they thought mathematics can be learnt by high intellectuals. But nowadays mathematics is the language that all form of disciplines adopt to communicate their ideas for example, Physics, Economics, Biology and others. The economics theories which concern human behavior cannot be analyzed effectively and predict good future mathematics is ignored. Therefore, the role of mathematics in everyday life cannot be overemphasized because mathematics trains the mind and helps in studying other discipline.

In view of these, mathematics has remained as a core subject on the timetable of pre-tertiary schools in Ghana and has been of immense interest to many students who take it as elective subject. However, it is very disheartening to hear most students complain that Mathematics is difficult. This complains perhaps can be attributed to the fact that most of the mathematics teachers in our senior secondary schools have a scrappy knowledge about the subject matter or content. In this regard, they are unable to add the least flesh to mathematics, which will make the majority who have difficulty in learning the subject love to dance with it. Another great setback the subject suffers is the use of inefficient methods in presenting the subject matter to students. Perhaps, more efficient methods of treating topics in mathematics are needed in order to save the situation.

The solution of quadratic equation has been of interest not only to contemporary scholars, but it has been important long before the birth of Christ. The Babylonians and the Egyptians, who were thought to have started mathematical concepts in solving problems and practical application of algebra to real life situation used the quadratic equation to solve problems in areas like profit and loss. According to Kramer 1981, this was due to the pricing system in Mesopotamia at that time. From that time, the quadratic equation has been of immense interest in various fields of human endeavour.

In Ghanaian education system, the Senior High School (SHS) curriculum required the solution of quadratic equation for immediate use in learning other subjects like physics and elective mathematics in the first year. In spite of these applications of quadratic equation in the early years of SHS programs, the curriculum planners of the SHS Mathematics programs has delayed the solution of such equation until the third year (SHS Mathematics Core syllabus CRDD 1992).

In view of these, the physics and elective mathematics teachers in the first year are compelled to teach the solution of quadratic equation perfunctorily. As a result a very large proportion of students are unable to learn the solution of quadratic equation by formula, with relational understanding. This has lead to wrong attitude of students to the topic in particular and to Mathematics in general.

It has been observed that, not all quadratic equations can be solved by method of factorization. In fact, according to Lial and Miller, (1979) "Not all quadratics equation can be solved by this kind of factorization (Mathematics with application in Management, Natural and Social Science p23). This is particularly so when the quadratic equation is a non – square trinomial.

These conventional methods of solving quadratic equations have inherent limitations that tend to affect their teaching and learning. So this study presents the feasibility of a more powerful method, devoid of these weaknesses in the conventional method of solving quadratic equations.

The conventional methods of solving quadratic equations required a lot of pre-requisite. This is the reason why educators feel that it must be taught gradually by breaking it into bits trying to go about each bit gingerly in order not to jeopardize the interest of students. Because of the inherent drawbacks, teachers and writers tend to be inhibited and they try not to introduce quadratic expressions where **a** and **c** are large and have several factors between them.

As a result, when more powerful method of solving quadratic equations is available, there will be no fear about giving equation or exercises no matter the nature of the coefficients. One of these non – conventional method that is free from the inherent defects of the conventional method of solving quadratic equations is the *equivalent simultaneous linear equations* developed by Gyening and Wilmon (1999). This method uses pre –requisite concepts and skills normally taught at the basic level. These are properties of numbers, and finding the highest common factor of two numbers and finding the value of an expression when given the values of the variables involved, (Junior High Schools, JHS 2). It is, therefore, necessary to investigate whether this method can be efficiently taught at SHS1 as an alternative method of solving quadratic equation.

The inherent limitations of the conventional methods of solving quadratic equations taught in the first year of the SHS1 mathematics program have led to the problem being investigated. The purpose of the study, therefore, is to investigate if it will be feasible to teach the equivalent simultaneous linear equations for students in SHS1 to learn it with ease just as they do with other topics in their Mathematics (Core) syllabus.

II. OBJECTIVE

The objective of the study is to verify the feasibility of teach and learning the equivalent simultaneous linear equations in solving quadratic equation in SHS1.

Hypothesis

To guide the study, the following hypotheses will be tested at 5 per cent level of significance immediately following the treatment.

H_0: There will be no significant difference between the mean scores of both groups on the tests.

H_1: There will be significant difference between the mean scores of both groups on the tests.

H_0: Three weeks after the treatment has ended, there will be no significant difference between the mean scores of both groups on a test.

H_1: Three weeks after the treatment has ended, there will be significant difference between the mean scores of both groups on a test.

Limitations of the Study

As in most research activities, this study was influenced by the following condition: The method of solving quadratic equation by equivalent simultaneous linear equations is not in the SHS Core Mathematics syllabus and these tests taken will not be part of their normal class work, therefore, students might have put little premium on the treatment knowing that their performance will not have any effect on their continuous assessment grade.

Significance of the Study

Teachers of mathematics in SHS complain of many topics in mathematics syllabus with little time to teach them. Students complain of difficulty in learning and remembering the methods of solving quadratic equations. Also the West Africa Examination Council (WAEC) has been worried for some time now about the persistent errors in the solution of quadratic equations. All these are signals to mathematics educators to find new ways that could solve these problems facing teachers, students and the WAEC. It is in this vine that the importance of this study lies. If the objectives of this study are upheld at Akro Secondary Technical School, in Eastern Region, Ghana, then the stage will be set for a more extensive research to confirm or reject the findings. Also, it will make curriculum developers review the grade placement of the topic "solution of quadratic equations" in the SHS core mathematics syllabus for its optimum application in the study of other subjects. The study is also to find out if the use of the equivalent simultaneous linear equations can help to cut down the length of time during which students have to wait to be introduced to the solution of quadratic equations. It is thus to find out the effectiveness or otherwise of the method and make recommendations accordingly.

THE EQUIVALENT SIMULTANEOUS LINEAR EQUATIONS (ELSE)

Gyening and Wilmon (1999) developed the method of solving quadratic equation by equivalent simultaneous linear equations (ELSE) in an attempt to overcome the inherent weaknesses of the conventional methods of solving quadratic equations. The proponents claimed that the equivalent simultaneous linear equations can be used to solve quadratic equations more easily and quickly. The method is described as follows:

Gyening and Wilmon (1999) claimed that the quadratic expression $ax^2 + bx + c = 0$ can be reduced into equivalent system of linear equations;

$$ax_1 + ax_2 = -b$$
$$ax_1 - ax_2 = d$$

Where x_1 and x_2 are the roots of the quadratic equation and $d = -(b^2 - 4ac)$.

Thus, this simultaneous linear equation can be solved to obtain the solution set of the quadratic equation. The method, therefore, entails the following steps:

Step 1: Compute $d = -\sqrt{b^2 - 4ac}$

Step 2: Write down the ELSE i.e. $\begin{aligned} ax_1 + ax_2 &= -b \\ ax_1 - ax_2 &= d \end{aligned}$

Step 3: Solve the ELSE to obtain the values of x_1 and x_2

Step 4: Write down the solution set, $\{x_1, x_2\}$

Proof

Let x_1 and x_2 be the roots of the quadratic equation $ax^2 + bx + c = 0$ -----------(1)

Then, according to the well known elementary properties of quadratic equations;

$x_1 + x_2 = -b/a$ ------------------ (2)

and $x_1 x_2 = c/a$ -----------------------(3)

from equation (2) $ax_1 + ax_2 = -b$ -----------------(4)

From equation (3) $a^2 x_1 x_2 = ac$ ------------------(5)

Squaring equation (4) $a^2(x_1 + x_2)^2 = b^2$ -------------------------(6)

Now it is easy to show that $a^2(x_1 - x_2)^2 = a^2(x_1 + x_2)^2 - 4a^2 x_1 x_2$ --------------(7)

By making use of the (5), (6) and (7) we obtain $a^2(x_1 - x_2)^2 = b^2 - 4ac$ ----------------(8)

Finding the square root of both sides of (8) we get $a(x_1 - x_2) = -\sqrt{b^2 - 4ac}$ -------------(9)

Replacing $-\sqrt{b^2 - 4ac}$ in equation (9) by d, gives $ax_1 - ax_2 = d$ ----------(10)

Equations (4) and (10) constitute the equivalent simultaneous linear equations (ELSE)

The procedure is illustrated with a specific example below.

1. Solve $2x^2 - 9x - 5 = 0$.

Solution

Step 1: Compute $d = -\sqrt{b^2 - 4ac} = -\sqrt{(9)^2 - 4(2)(-5)} = -11$

Step 2: Write down the ELSE: $\begin{aligned} 2x_1 + 2x_2 &= -(-9) \\ 2x_1 - 2x_2 &= -11 \end{aligned}$ simplified as $\begin{aligned} 2x_1 + 2x_2 &= 9 \quad ----(1) \\ 2x_1 - 2x_2 &= -11 ----(2) \end{aligned}$

Step 3: Solve the ELSE to obtain the values of x_1 and x_2 ;

Adding (1) and (2); $4x_1 = -2$; $x_1 = -½$. Putting $x_1 = -½$ into (1); $2(-½) + 2x_2 = 9$; $x_2 = 5$.

Step 4: Write down the solution set, {- ½, 5}

2. Solve $48x^2 - 86x + 35 = 0$

Solution

Step 1: Compute $d = -\sqrt{(-86)^2 - 4(48)(35)} = -\sqrt{676} = -26$

Step 2: Write down the ELSE: $\begin{aligned} 48x_1 + 48x_2 &= -(-86) \\ 48x_1 - 48x_2 &= -26 \end{aligned}$ simplified as $\begin{aligned} 48x_1 + 48x_2 &= 86 -----(1) \\ 48x_1 - 48x_2 &= -26 ----(2) \end{aligned}$

Step 3: Solve the ELSE to obtain the values of x_1 and x_2 ;
Adding (1) and (2); $96x_1 = 60$; $x_1 = 60/96 = 5/8$. Putting $x_1 = 5/8$ into (1); $48\left(\frac{5}{8}\right) + 48x_2 = 86$; $x_2 = 86 - 30 = 7/6$.

Step 4: Write down the solution set, {5/8, 7/6}

Advantages of the ELSE

According to Gyening and Wilmon (1999), the equivalent simultaneous linear equations has the following advantages over the conventional methods of solving quadratic equations:

The concepts entailed in the derivation of ELSE are simple enough to be understood by the average ability pupil in the first year of the secondary school. One of the concepts is the relationship between the roots and the coefficients of quadratic equations. According to Cooney et al (1975, p. 159-160) this can be taught in a single lesson by using the inductive discovery strategy. The other pre-requisite concept is the property of real number expressed symbolically as $(p-q)^2 = (p+q)^2 - 4pq$ where p and are real numbers.

Again, unlike the graphical method, it is efficient and effective. In contrast to the complexity of the method of completing of squares, the methods of ELSE is simple and by virtue of its inherent advantages, it has the potential of being easier to teach and easier to use. In this regard, it can be presumed to be superior to the quadratic formula whose formidable structure, including the weird-looking double sign, has put many a student to flight on first encounter.

One other important educational value of this method is the student's increased awareness of the general mathematical method of solving a problem by replacing a given problem with a simpler equivalent to facilitate the solution (Polya, 1957; Fremont, 1969; Sawyer, 1975). This is the essence of what Skemp (1986) refers to as the principle of interchangeability which is implicit in the idea of equivalent class.

III. METHODOLOGY

The first year students who reported for the first term of 2009/10 academic year at Akro Secondary Technical School, Eastern Region of Ghana were sampled for the study. This school was chosen for convenience. There were 72 students who took part in this study and it consisted of 42 boys and 30 girls. For the selection into experimental and control groups the students were stratified by sex for equal representation. The boys were divided into two groups by systematic sampling as well as the girls. Each stratum was assigned **A** and **B**. A coin was tossed to determine the control group and group **B** was chosen as the control group. Both groups were informed about the study and experimental group was caution to use only the method thought during this study. Base on the sampling technique, it is expected that the average performance of both groups should be the same at 5% level of significance. However, if the average performance of the experimental group over the period is higher or better than that of the average performance of control group then the improvement in experimental group will be attributed to treatment given. The scores on the tests one to five were obtained during the treatment. To evaluate whether the skill taught could be applied effectively by the students both groups took test six which was obtained three weeks after the treatment to see if students from each group could remember and use what they have learnt.

Six achievement tests were used as instruments for collecting the data. The test one to two consisted of three items each, test three consists of four items and test four, five and six consisted of five items each. The scores on the respective tests were then converted to twenty percentage scale. The items on the test were selected from SHS 1-3 Mathematics (core) textbook and the researcher, on the other hand, constructed some of the test to increase the difficulty level. All test were discussed with other mathematics tutors in respect of content and face validity. The scoring of the tests was done manually. Each item was broken into steps and partial credits were awarded

The reliability coefficients of the tests were determined by the Cronbach Alpha method. This method was chosen because partial credits were awarded in scoring the responses. The Cronbach Alpha method is given as:

$r_{11} = \frac{n}{n-1}\left(\frac{\sigma_t^2 - \sum_{i=1}^{n}\sigma_i^2}{\sigma_t^2}\right)$, *wher* r_{11} *= coefficient of reliability,* n *= number of items,* σ_t^2 *= variance of the entire test and* σ_i^2 *= variance of the i-th item.*

Wilcoxon Rank-Sum Test was the test statistic was used to compare the performances. It is given as:

$$Z = \frac{T - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}}$$

, where T = sum of the ranks with less frequent sign and N = Sample size.

IV. RESULTS AND DISCUSSION

The ages of the students in the sample ranged from 15 to 21 years with average age of 16 years. The students mostly had their Junior High Schools (JHS) education in rural and deprived schools in Ghana. The results of most of the students at JHS were grade 6 in English, Science or Mathematics. Most of their parents were petty traders and food crop farmers with basic or no formal education.

Test Reliability

The result of Cronbach Alpha reliability coefficients statistics is shown in table 1 below. The reliability coefficient of the test ranges from 72 to 95 percent. From the table, test **I** and **II** had three questions with test reliability coefficients of 0.876 and 0.9478, respectively. Test **III** had four questions with test reliability coefficients of 0.7199 while test **IV**, **V** and **VI** had five questions each with test reliability coefficients of 0.7795, 0.8321 and 0.8295, respectively. From the table, all the test reliability coefficients were above 70 percent; this shows that to a large extent the test had measured what it intended to measure. Therefore, all the six tests were valid and reliable.

Table 1: The Cronbach Alpha Reliability Coefficients Statistics

	n	σ_i^2	σ_1^2	σ_2^2	σ_3^2	σ_4^2	σ_5^2	$\sum \sigma_i^2$	r_{11}
Test I	3	61.7446	7.1637	10.1891	8.3333			25.6862	0.8760
Test II	3	22.3474	3.1254	3.0929	2.0078			8.2262	0.9478
Test III	4	17.1639	2.2808	1.9068	2.0977	1.6116		7.8968	0.7199
Test IV	5	38.5187	2.8370	2.9702	2.9739	2.5559	3.1626	14.4996	0.7795
Test V	5	42.8966	2.7645	2.7574	3.0476	3.1106	2.6620	14.3420	0.8321
Test VI	5	42.5660	2.7255	2.7206	3.0030	3.0604	2.8091	14.3186	0.8295

Comparison of Groups Performance

The descriptive statistics in table 2 below shows the summary of the performance of both groups in the study. From the table, in test **I** the experimental group on average scored 6.5 out of the total score of 20 with standard deviation of 6.3 and median score of 5.3 while control group on average scored 5.2 out of the total score of 20 with standard deviation of 6.4 and median score of 2.5. In test **II** the experimental group on average scored 10 out of the total score of 20 with standard deviation of 6.1 and median score of 13.3 while control group on average scored 7.6 out of the total score of 20 with standard deviation of 5.3 and median score of 8. In test **III** the experimental group on average scored 9.1 out of the total score of 20 with standard deviation of 6.9 and median score of 9.3while control group on average scored 8.9 out of the total score of 20 with standard deviation of 6.6 and median score of 8. In test **IV**, the experimental group on average scored 11.2 out of the total score of 20 with standard deviation of 5.6 and median score of 11.5 while control group on average scored 6.6 out of the total score of 20 with standard deviation of 5.9 and median score of 5. In test **V** the experimental group on average scored 13.9 out of the total score of 20 with standard deviation of 5.3 and median score of 15 while control group on average scored 6.9 out of the total score of 20 with standard deviation of 5.8 and median score of 7.3. In test **VI** the experimental group on average scored 11.1 out of the total score of 20 with standard deviation of 5.7 and median score of 11 while control group on average scored 7.3 out of the total score of 20 with standard deviation of 5.8 and median score of 7. From the table, the average performance of both groups in test **I - III** seems to be equal but the median marks for the experimental group are higher. As the test became more difficult in **IV – VI**, both average and median marks of the experimental group seems to be superior. But these differences in performance are they statistically significant? The Wilcoxon Rank-Sum Test is used to test statistical significance of the differences in performance.

Table 2: Comparative Performance in the Tests over Time

Groups Performance	N	Mean	Standard deviation	Minimum	Maximum	Median
Experimental 1	36	6.52	6.29	0	16	5.33
Experimental 2	36	10	6.06	0	19	13.33
Experimental 3	36	9.1	6.89	0	20	9.3
Experimental 4	36	11.19	5.64	0	20	11.5
Experimental 5	36	13.88	5.27	0	20	15
Experimental 6	36	11.08	5.67	0	20	11
Control 1	36	5.22	6.38	0	18	2.5
Control 2	36	7.61	5.31	0	17	8
Control 3	36	8.92	6.62	0	20	8
Control 4	36	6.55	5.92	0	20	5
Control 5	36	6.85	5.79	0	20	7.3
Control 6	36	7.25	5.8	0	20	7

The result of Wilcoxon Rank-Sum Test were summarised in table 3 below. From the table, in test **I** the experimental group had 16 scores higher while the control group had 13 scores higher. The control group had the lowest rank sum with z-value of -0.898 with p-value of 0.369 which is greater than 0.05. This means that the differences in performance of both groups are not statistically significant. Therefore, the performance of both groups is the same. Considering test **II**, the experimental group had 19 scores higher while the control group had 16 scores higher. The control group had the lowest rank sum with z-value of -0.016 with p-value of 0.987 which is greater than 0.05. This means that the differences in performance of both groups are not statistically significant. Therefore, the performance of the control and experimental groups are the same. The experimental group in test **III** had 19 scores higher while the control group had 13 scores higher. The control group had the lowest rank sum with z-value of -1.619 with p-value of 0.106 which is greater than 0.05. This means that the differences in performance of both groups are not statistically significant. Therefore, the performance of the control and experimental groups are the same. In test **IV**, the experimental group had 24 scores higher while the control group had 7 scores higher. The control group had the lowest rank sum with z-value of -2.825 with p-value of 0.005 which is less than 0.05. This means that the differences in performance of both groups are statistically significant. Therefore, the performance of the experimental group in test IV is better as compared to the performance of the control group. In test **V**, the experimental group had 23 scores higher while the control group had 6 scores higher. The control group had the lowest rank sum with z-value of -3.894 with p-value of zero which is less than 0.05. This means that the differences in performance of both groups are statistically significant. Therefore, the performance of the experimental group in test V is superior.

From the results, it was clear that the performance of the experimental group was better than that of the control group in test IV to V. As the test items were progressively difficult, the performance of the control group declined while the performance of the experimental group progressively improved. This performance uphold the alternative hypothesis that immediately following the treatment, there is significant difference between the mean scores of the experimental group and the control group on the tests.

Finally, in test VI, the experimental group had 23 scores higher while the control group had 8 scores higher. The control group had the lowest rank sum with z-value of -2.207 with p-value of 0.027 which is less than 0.05. This means that the differences in performance of both groups are statistically significant. This performance uphold the alternative hypothesis that three weeks after the treatment has ended, there is significant difference between the mean scores of the experimental group and the control group on a test. Therefore, the performance of the experimental group in test VI is superior. This showed that students in the experimental group learn and remember the equivalent simultaneous linear equations and applied it more than they counterpart in the control group.

Table 3: The Wilcoxon Rank-Sum Test Results for both Experimental and Control Group

		N	Mean Rank	Sum of Ranks	Z	prob
Experimental Test 1 - Control Test 1	Negative Ranks	13	13.54	176	-0.898	0.369
	Positive Ranks	16	16.19	259		
	Ties	5				
	Total	34				
Experimental Test 2 - Control Test 2	Negative Ranks	16	19.63	314	-0.016	0.987
	Positive Ranks	19	16.63	316		
	Ties	0				
	Total	35				
Experimental Test 3 - Control Test 3	Negative Ranks	13	13.65	177.5	-1.619	0.106
	Positive Ranks	19	18.45	350.5		
	Ties	2				
	Total	34				
Experimental Test 4 - Control Test 4	Negative Ranks	7	14.86	104	-2.825	0.005
	Positive Ranks	24	16.33	392		
	Ties	2				
	Total	33				
Experimental Test 5 - Control Test 5	Negative Ranks	6	6.25	37.5	-3.894	0
	Positive Ranks	23	17.28	397.5		
	Ties	1				
	Total	30				
Experimental Test 6 - Control Test 6	Negative Ranks	8	16.94	135.5	-2.207	0.027
	Positive Ranks	23	15.67	360.5		
	Ties	1				
	Total	32				

V. CONCLUSION

This study compared the performance of students who were taught new concept that is solving quadratic equation by method of equivalent simultaneous linear equations in experimental group and the control group who were taught the conventional method of solving quadratic equation. Both groups were given the same instructional hours and took the same test items. The results showed that, at 5 per cent level of significance, the experimental group performance was superior in test IV and V. In terms of remembering the concept after three weeks the performance of the experimental group in test VI was also superior to that of the control group. In sum, it is clearly shown that students learnt the method of equivalent simultaneous linear equations well, able to remember and applied it better than their friends in the control group who used the conventional method. So the method of equivalent simultaneous linear equations has the power that overcomes the inherent problems in the conventional methods. Therefore, a nationwide study is recommended to verify the feasibility of the method of equivalent simultaneous linear equations and subsequent inclusion in the core mathematics curriculum to help solved the problem of solving quadratic equation in Ghana, Africa and the world at large.

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