

Learning to Design Geometer's Sketchpad Activities for Teaching Mathematics through Lesson Study

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ABSTRACT

This study aimed to develop pre-service teachers' skills and confidence to design Geometer's Sketchpad (GSP) activities for teaching mathematics through Lesson Study (LS). The researcher employed a case study research design and the participants comprised 26 pre-service secondary teachers who enrolled in the mathematics teaching methods course in a Malaysian public university. Thirteen LS groups each comprising 2 participants were set up in two tutorial groups. Qualitative data were collected for each LS group through observations, written lesson plans, reflections and GSP activities. This paper discusses the changes in the GSP activities of one of the LS groups for teaching the sum of the interior angles of a polygon in the first, second and third lessons. The findings indicate that the participants of this LS group showed gradual improvement in their skills and confidence to design GSP activities for teaching the topic after engaging in LS.

Keywords: The Geometer's Sketchpad (GSP), Lesson Study, Pre-service secondary teachers

I. INTRODUCTION

The Geometer's Sketchpad (GSP) is a dynamic geometry software program for constructing and investigating mathematical objects that enhances the teaching and learning of geometry and many other areas of mathematics. In Geometry, for example, students can use GSP to construct a polygon and then investigate the sum of the interior angles of the polygon. In other areas of mathematics such as Algebra and Trigonometry, students can employ GSP to construct a quadratic function and a trigonometric function and then investigate the properties of the functions respectively (Chanan, 2000).

In fact, a substantial number of studies have indicated that GSP is an essential tool for enhancing students' learning of mathematics. For example, (i) mathematics achievement and time of independent investigation using GSP were significant predictors of conjecture-making ability (Elchuck, 1992); (ii) the abilities to conjecture and justify conjectures in a geometry class using GSP were directly related to proof-writing abilities (Frerking, 1995); (iii) learning mathematics using GSP enhanced students' geometry achievement (Nurul Hidayah Lucy, 2005), students' van Hiele levels of geometric thinking (Chew, 2007; Chew & Lim, 2010; Choi, 1996; Choi-Koh, 1999; July, 2001; McClintock, Jiang & July, 2002; Thompson, 2006) as well as secondary students' geometry achievement

and van Hiele levels of geometric thinking (Chew & Noraini Idris, 2006; Noraini Idris, 2007); (iv) most of the students also showed positive perceptions of using GSP to learn geometry (Chew & Noraini Idris, 2006; Noraini Idris, 2007); (v) learning mathematics using 'G-Reflect', a GSP-based courseware, had a significant effect on secondary students' achievement and motivation in learning the topic of 'Reflections' (Rosanini Mahmud, Mohd Arif Hj Ismail & Lim, 2009); (vi) the dynamic capability of GSP, inquiry-based tasks as well as student-student and researcher-student interactions deepened students' conception of two-dimensional shapes (Driskell, 2004); and (vii) learning mathematics using GSP enhanced pre-service secondary mathematics teachers' understanding of limits of sequences (Cory & Garofalo, 2011).

In view of its importance, the Malaysian Ministry of Education has purchased the GSP license and supplied the GSP software to all secondary schools since 2004 and it was envisaged that this initiative would benefit many students, teachers and teacher educators nationwide. Despite this initiative, teacher enthusiasm and willingness to use GSP remains an issue to be addressed (Teoh & Fong, 2005). A survey conducted by Kasmawati (2006) on 151 secondary mathematics teachers in the state of Penang showed that 26% of the teachers had attended GSP training courses but only 2% used GSP to teach mathematics in the classroom. The two main reasons given by the mathematics teachers for the low percentage of using GSP in the classroom were firstly lack of time to prepare a GSP sketch, and secondly lack of skills and confidence to use GSP to teach mathematics.

Therefore, there is a need to develop pre-service secondary teachers' skills and confidence to design GSP activities for teaching mathematics through a collaborative group effort such as Lesson Study (LS) which

will provide helpful support and sustain the continuous use of GSP in the mathematics classroom as advocated by the Malaysian Ministry of Education (2003).

II. PURPOSE OF THE STUDY

The purpose of this study was to develop pre-service secondary teachers' skills and confidence to design GSP activities for teaching mathematics through LS. More specifically, this paper aimed to address the following research question: What changes, if any, occurred in the participants' GSP activities of one of the LS groups for teaching the sum of the interior angles of a polygon in the first, second and third lessons after engaging in LS?

III. CONCEPTUAL FRAMEWORK

The researcher employed LS as the conceptual framework underpinning this study. LS is a direct translation for the Japanese term *jugyokenkyu* (*jugyo* means lesson and *kenkyu* means study or research) and it was already well established in Japan since the 1960s. Today, it is an on-going practice as a form of teacher professional development whereby teachers actively engage in a continuous process of improving the quality of their teaching and to enrich their students' learning experiences (Fernandez, & Yoshida, 2004). More specifically, LS is a process by which small groups of teachers meet at stipulated time to collaboratively plan lessons, observe these lessons unfold in actual classrooms, discuss their observations and to revise the lesson plans. According to Fernandez and Yoshida (2004), LS consists of six main steps: (1) collaboratively planning the lesson plan, (2) seeing the lesson plan in action, (3) discussing the lesson plan, (4) revising the lesson plan, (5) teaching the new version of the lesson, and (6) sharing reflections about the new version of the lesson.

The six steps of the LS process are discussed in more detail in the section on research procedure.

LS was chosen as the conceptual framework of this study because research has shown that it not only improves teachers' learning and supports teachers to grow professionally (Stigler & Hiebert, 1997, 1999; Shimahara, 1998; Lewis & Tsuchida, 1998; Yoshida, 1999; Lewis, 2000; Fernandez, & Yoshida, 2004; Lim, White & Chiew, 2005) but also it is a worthwhile and beneficial learning experience for pre-service teachers. Chiew and Lim (2003) found that LS helped improve pre-service mathematics teachers' pedagogical content knowledge and enhance their confidence to teach mathematics. Fernandez and Robinson (2006) identified three main categories as central to pre-service teachers' learning through LS, namely connecting theory and practice, collaboration, and reflection. Lim (2006) found that despite time constraint and peer conflict, the majority of pre-service secondary teachers agreed that LS was a good means of preparing them to teach mathematics and they would like to continue the LS process in schools after graduation. In addition, Chew and Lim (2011a) found that LS enhanced secondary school teachers' knowledge and skills of using GSP to teach the topics of Lines and Planes in Three Dimensions, Loci in Two Dimensions as well as Plans and Elevations. In particular, Chew and Lim (2011b) found that LS could enhance pre-service secondary teachers' skills of using GSP to teach the concept of Regular Polygons in the Malaysian Form Three Mathematics syllabus.

IV. MATERIALS AND METHOD

Research design and participants

The researcher employed a case study research design to examine the changes in the pre-service secondary mathematics teachers' skills and confidence to design GSP activities

in the first, second and third lessons after engaging in LS (Gall, Gall & Borg, 2003). The participants of this study comprised 26 pre-service secondary teachers who enrolled in a mathematics teaching methods course in a Malaysian public university.

Research procedure

In the first two-hour lecture, the researcher who is the coordinator of the course explained the course outline, the coursework and the LS process to all the participants. The coursework consisted of a review of a journal article on teaching mathematics with GSP by an individual participant, a 40-minute lesson plan for teaching mathematics with GSP by a LS group, and a simulated teaching of the planned 40-minute lesson using GSP by an individual member of a LS group. At the end of the lecture, the participants were divided into two tutorial groups. Thirteen LS groups each comprising 2 participants were set up in the two tutorial groups. There were six LS groups in the first tutorial group (known as LS Group 1 to LS Group 6) and seven LS groups in the second tutorial group (known as LS Group 1 to LS Group 7). Each tutorial group would meet at a specific tutorial time for one hour every week.

Next, the researcher conducted two GSP workshops during the first two tutorials for each tutorial group. In the first GSP workshop which was held during the first tutorial, the participants learnt the functions of the Title bar, Menu bar, Sketch plane, and Toolbox of GSP as well as how to use the basic tools of GSP such as Selection Arrow tool, Point tool, Compass tool, Straightedge tool, Text tool, and Custom tool to construct mathematical objects like points, segments, rays, lines, circles, and polygons. In the second GSP workshop which was held during the second tutorial, the participants learnt how to design a GSP activity for teaching the topic of Pythagoras' Theorem based on Benett's (1999)

GSP activity sheet. After the workshops, the participants were encouraged to learn more about GSP by referring to GSP books which were available in the library and other resources on the Internet.

After the workshops, the six main steps of LS were implemented during the subsequent tutorials as follows:

Step 1 (Collaboratively Planning the Lesson Plan)

This initial step was implemented during the third tutorial. Firstly, each LS group was allowed to choose a topic in the Malaysian secondary school mathematics syllabus. Secondly, the members of each LS group collaboratively developed a 40-minute lesson plan for teaching the chosen topic using GSP. Finally, each LS group planned subsequent meetings outside the lecture and tutorial schedule to complete their lesson plan and GSP activities before the fourth tutorial.

Step 2 (Seeing the Lesson Plan in Action)

For each tutorial group, one member from LS Group 1 (Teacher 1) taught the 40-minute lesson as planned using GSP version 4.05M installed in his/her laptop and a mounted LCD projector to their peers (students) in the Mathematics Teaching Room during the fourth tutorial. The students also used GSP version 4.05M installed in their laptop to construct the GSP sketches. His or her partner of LS Group 1 (Teacher 2) and the researcher observed the lesson using the lesson plan and GSP activities to guide their observations.

Step 3 (Discussing the Lesson Plan)

After the lesson that is about twenty minutes before the end of the tutorial, the peers and the researcher provided comments and suggestions to help the members of LS

Group 1 improve their lesson plan and GSP activities.

Step 4 (Revising the Lesson Plan)

After the tutorial, the members of LS Group 1 in each tutorial group planned subsequent meetings outside the lecture and tutorial schedule to revise their lesson plan and GSP activities according to their peers' and the researcher's comments and suggestions as well as their own observations before the fifth tutorial. The end product of this step would be a revised lesson plan and GSP activities.

Step 5 (Teaching the New Version of the Lesson)

During the fifth tutorial, the revised lesson was then taught by the other partner of LS Group 1 (Teacher 2) using GSP version 4.05M installed in his/her laptop and a mounted LCD projector to different peers (students) in the other tutorial group. The students also used GSP version 4.05M installed in their laptop to construct the GSP sketches. His or her partner of LS Group 1 (Teacher 1) and the researcher observed the lesson using the revised lesson plan and GSP activities to guide their observations. After the lesson that is about twenty minutes before the end of the tutorial, the peers and the researcher provided comments and suggestions to further improve the lesson plan and GSP activities.

Step 6 (Sharing Reflections about the New Version of the Lesson)

After the tutorial, the members of LS Group 1 in each tutorial group planned subsequent meetings to revise their lesson plan and GSP activities for a second time according to their peers' and the researcher's comments and suggestions as well as their own observations before the sixth tutorial. The end product of this step would be a final lesson

plan and GSP activities for submission to the researcher as their coursework during the next tutorial.

Steps 2 to 6 were repeated for the other LS Groups in the subsequent tutorials accordingly. For each LS group, qualitative data were collected through observations, written lesson plans, reflections and GSP activities.

V. RESULTS AND DISCUSSION

In this paper, the discussion of the findings focuses on the analysis of the GSP activities for teaching the sum of the interior angles of a polygon in Malaysian Form 3 Mathematics in the first, second and third lessons of one of the selected LS groups. The LS group consisted of two female participants. The learning objective

of the lesson was to enable students to understand that the sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$. The changes in the participants' GSP activities in the first, second and third lessons after engaging in LS are presented and discussed in the following sections respectively.

First lesson

In the first lesson, the LS group designed a GSP activity to enable students to understand that the sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$. Firstly, the students were asked to construct a pentagon by following the instructions in the GSP activity sheet.

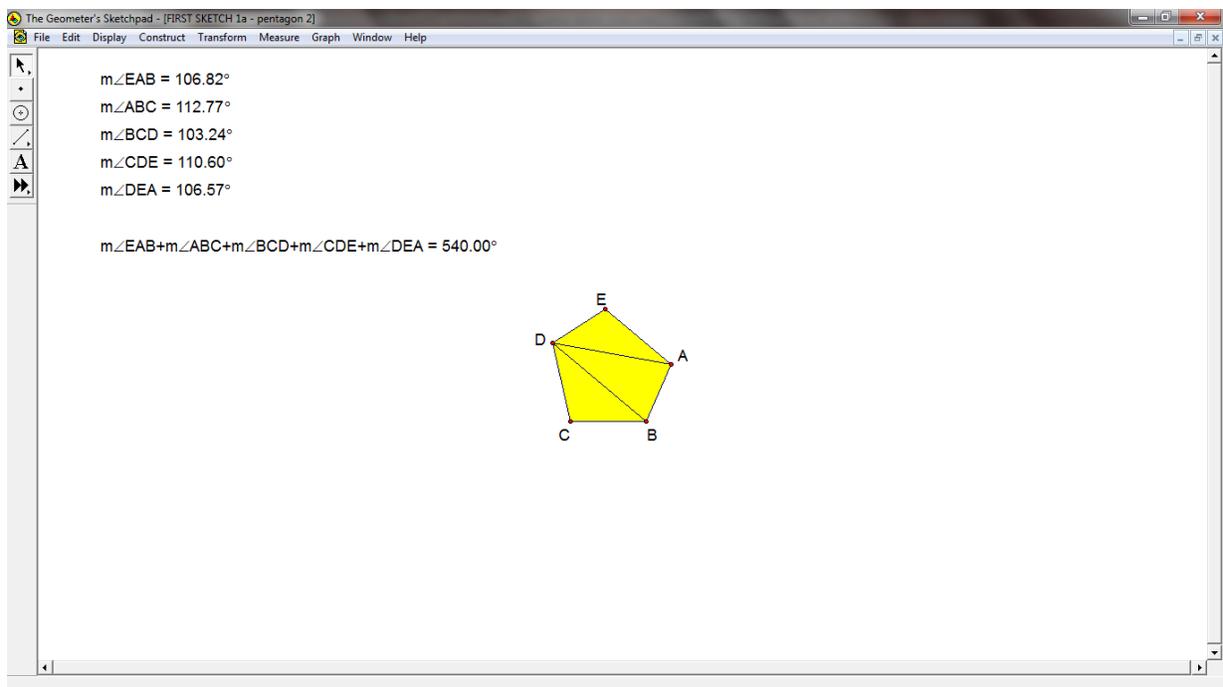


Figure 1a. GSP sketch of a pentagon

The students: (1) constructed a circle using the Compass Tool; (2) constructed four points on the circle using the Point Tool; (3) labelled all the points on the circle as A, B, C, D and E in a clockwise direction using the Text Tool;

(4) constructed five segments, namely AB, BC, CD, DE and EA using the Straightedge Tool; (5) constructed the pentagon interior using the Pentagon Interior command in the Construct menu; (6) hid the circle using the

Hide Circle command in the Display menu; (7) constructed two diagonals, namely AD and BD using the Straightedge Tool; (8) measured the five interior angles using the Angle command in the Measure menu; and (9) calculated the sum of the interior angles using the Calculate command in the Measure menu. Figure 1a shows the GSP sketch of the LS group and most of the students managed to construct the GSP sketch.

Secondly, the students were asked to construct a regular hexagon by following the instructions in the GSP activity sheet. The students: (1) constructed a circle using the Compass Tool; (2) labelled the points on the circle as O and A using the Text Tool; (3) constructed six congruent circles with points A, B, C, D, E and F as the centres of the

respective circles using the Circle By Center+Point command in the Construct menu; (4) constructed six segments, namely AB, BC, CD, DE, EF and FA using the Straightedge Tool; (5) constructed the hexagon interior using the Hexagon Interior command in the Construct menu; (6) hid the circle using the Hide Circle command in the Display menu; (7) constructed three diagonals, namely BF, CF and DF using the Straightedge Tool; (8) measured the six interior angles using the Angle command in the Measure menu; and (9) calculated the sum of the interior angles using the Calculate command in the Measure menu. Figure 1b shows the GSP sketch of the LS group and the majority of the students managed to construct the GSP sketch as well.

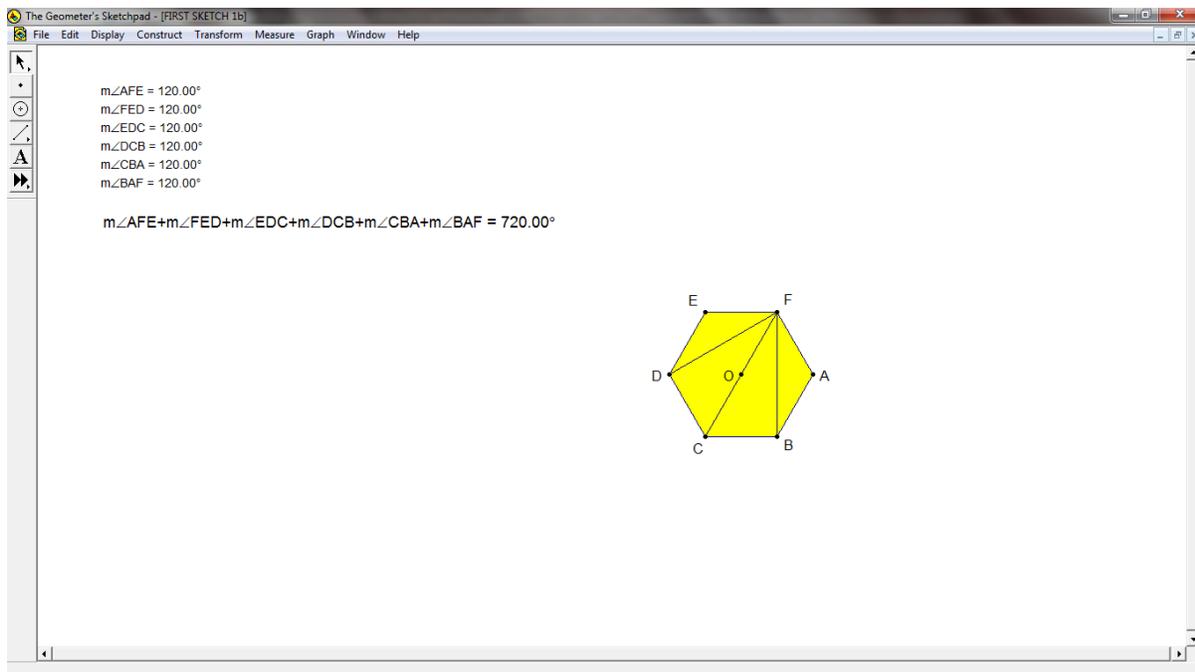


Figure 1b. GSP sketch of a regular hexagon

In addition, the students were asked to click and drag point A of the pentagon and regular hexagon and observe the measurements of all the interior angles and the sum of all the interior angles. Then, they had to record their observations in a table as shown

in Table 1. Based on their findings in Table 1, the students were asked to make a conclusion for the sum of the interior angles of a triangle, quadrilateral, pentagon and hexagon. Lastly, they were asked to make a conclusion for the sum of the interior angles of a polygon.

TABLE 1
The table provided in the GSP activity sheet

Number of sides	Number of Triangles	Sum of the Interior Angles	Sum of the Interior Angles (from your GSP Sketch)
3	1	$1 \times 180^\circ$	
4			
5			
6			

After the first lesson some of their peers commented that students ought to be given the opportunities to construct a triangle and quadrilateral so that they could investigate the sum of the interior angles of a triangle and quadrilateral which in turn would help them to make the correct conclusion. Besides, they also commented that the construction of a regular hexagon and investigating the sum of the interior angles of the regular hexagon was not so suitable because it is a type of regular polygon and thus it does not represent all hexagons. So, they suggested that an irregular hexagon ought to be constructed by students to investigate the sum of the interior angles which in turn would assist them to draw the correct conclusion. Additionally, some of their peers suggested that students ought to construct the polygons in a single GSP sketch instead of separate GSP sketches so that students could easily observe the relationship between the number of triangles in a polygon and the sum of the interior angles of the polygon.

Further, to help students conclude that the sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$, the researcher suggested that students ought to be given the opportunities to construct as many examples of polygons as possible starting from a triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon and n -gon so that they could easily observe the relationship between the number of sides of a polygon and

the number of triangles in a polygon as well as the relationship between the number of triangles in a polygon and the sum of the interior angles of a polygon. However, a more suitable table ought to be provided so that students could record their observations and easily observe the relationships. The suggested table ought to contain the name, number of sides, number of triangles and the sum of the interior angles of the polygons.

After the tutorial, the LS group members were required to make changes to their first GSP activity based on the comments and suggestions given by their peers and the researcher as well as their own observations. Further, they were advised to do further readings on the topic and GSP by referring to GSP books in the library and other resources on the Internet.

Second lesson

Based on the peers' and researcher's comments and suggestions, the LS group members revised their GSP activity in the second lesson by referring to the Malaysian Form 3 Mathematics textbook and GSP books such as *Exploring Geometry with The Geometer's Sketchpad* (Bennett, 1999) and *Geometric Activities for Middle School Students with The Geometer's Sketchpad* (Wyatt, Lawrence, & Foletta, 1999). Additionally, the LS group members also

sought help and guidance from the researcher to revise the GSP activity.

In the second lesson, the students were asked to construct a triangle, quadrilateral, pentagon and hexagon in a single GSP sketch by following the instructions in the revised GSP activity sheet. The students: (1) constructed a triangle using the Straightedge Tool and labelled the vertices as A, B and C using the Text Tool; (2) constructed a quadrilateral using the Straightedge Tool and labelled the vertices as A, B, C and D using the Text Tool; (3) constructed a pentagon using the Straightedge Tool and labelled the vertices as A, B, C, D and E using the Text Tool; (4) constructed a hexagon using the Straightedge Tool and labelled the vertices as A, B, C, D, E and F using the Text Tool; (5) measured all the interior angles of the polygons using the Angle command in the

Measure menu; (6) calculated the sum of the interior angles of the polygons using the Calculate command in the Measure menu; and (7) constructed all the diagonals of the polygons using the Straightedge Tool. Figure 2 shows the GSP sketch of the LS group and most of the students managed to construct the GSP sketch.

Further, the students were asked to click and drag any vertex of the polygons and observe the number of sides, number of triangles and the sum of the interior angles of the polygons. Next, they had to record their observations in a revised table as shown in Table 2. Based on the relationship among the number of sides of a polygon, the number of triangles in a polygon and the sum of the interior angles of a polygon in Table 2, the students were asked to make a conclusion about the sum of the interior angles of a polygon.

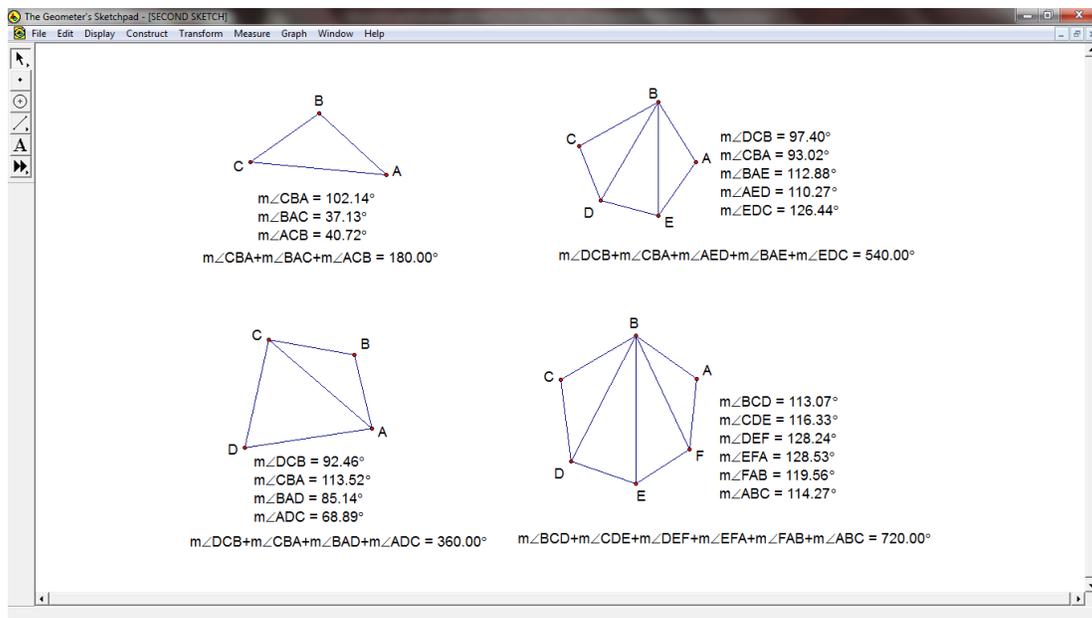


Figure 2. GSP sketch of the second lesson

TABLE 2
The revised table provided in the GSP activity sheet

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angles
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			
n-gon			

After the second teaching, most of their peers gave positive comments on the revised GSP activity such as students were given the opportunities to construct the polygons with increasing number of sides in a single GSP sketch and then investigate the sum of the interior angles of the polygons which helped the students to observe the relationship among the number of sides, number of triangles and the sum of the interior angles of a polygon. Nevertheless, they suggested that students ought to be given the opportunities to construct other polygons listed in Table 2 to help them observe the relationship more clearly so that they could conclude that the sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$.

In addition, the researcher suggested that students ought to be provided with guided questions at the end of the activity so that they could make the correct conclusion based on firstly, the relationship between the number of sides of a polygon and the number of triangles in a polygon and secondly, the relationship between the number of triangles in a polygon and the sum of the interior angles of a polygon.

After the tutorial, the LS group members were required to make final changes to their second GSP activity based on the comments and suggestions given by their peers and the researcher as well as their own observations. They were also advised to do further readings on the topic and GSP.

Third lesson

Based on their peers' and the researcher's comments and suggestions, the LS group revised their GSP activity accordingly by referring to the Malaysian Form 3 Mathematics textbook and the above GSP books as well as seeking further help and guidance from the researcher. As a result, they successfully revised the GSP activity as evidenced in the GSP activity sheet.

In the revised GSP activity sheet, students were first asked to construct a triangle, quadrilateral, pentagon and hexagon in a single GSP sketch by following the instructions in the revised GSP activity sheet: (1) construct a triangle, quadrilateral, pentagon and hexagon using the Straightedge Tool and labelled the vertices according to the vertices

of the respective polygons using the Text Tool; (2) measure all the interior angles of the polygons using the Angle command in the Measure menu; (3) calculate the sum of the interior angles of the polygons using the Calculate command in the Measure menu; and (4) construct all the diagonals of the polygons using the Straightedge Tool. The resulting GSP sketch will be the same as the one in the second lesson (see Figure 2). Next, students were asked to construct a heptagon, octagon, nonagon, and decagon in another GSP sketch by following the instructions in the revised GSP activity sheet: (1) construct a heptagon, octagon, nonagon, and decagon using the Straightedge Tool and labelled the vertices according to the vertices of the respective polygons using the Text Tool; (2) measure all

the interior angles of the polygons using the Angle command in the Measure menu; (3) calculate the sum of the interior angles of the polygons using the Calculate command in the Measure menu; and (4) construct all the diagonals of the polygons using the Straightedge Tool. The end product will be a GSP sketch of a heptagon, octagon, nonagon, and decagon as shown in Figure 3.

In the revised GSP activity sheet, students were also asked to click and drag any vertex of the polygons and observe the number of sides, number of triangles and the sum of the interior angles of the polygons. Next, they had to record their observations in a revised table which is the same as the previous one (see Table 2).

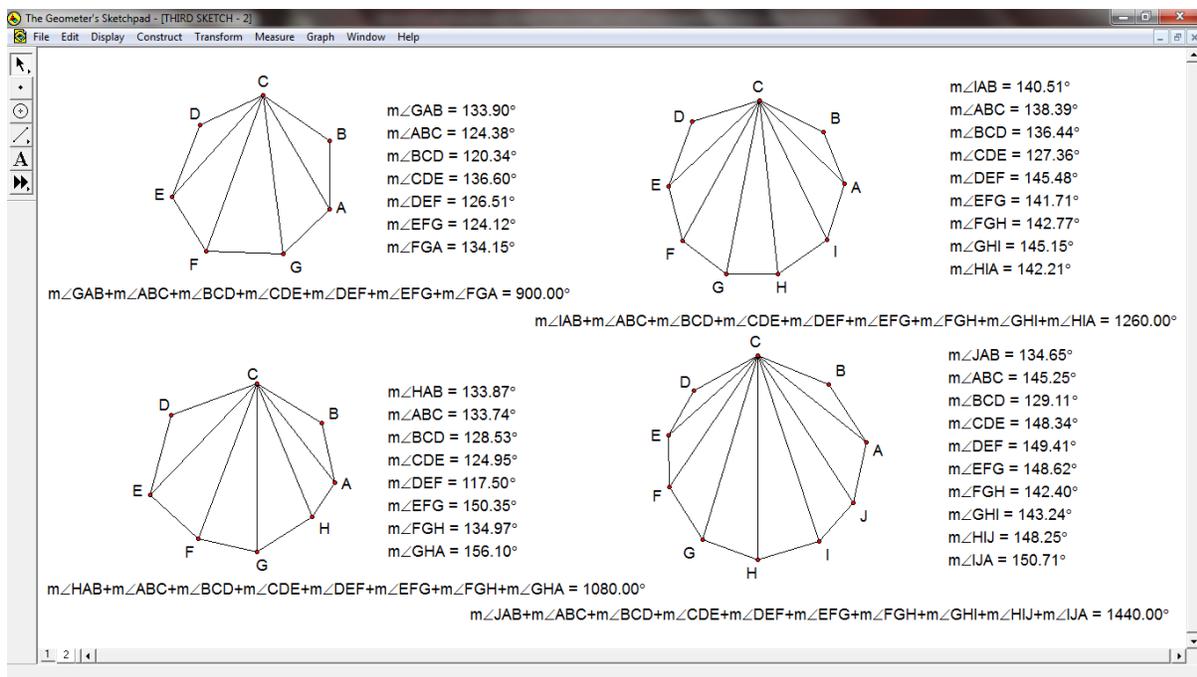


Figure 3. GSP sketch of a heptagon, octagon, nonagon, and decagon in the third lesson

In addition, students were provided with guided questions at the end of the activity so that they could make the correct conclusion based on firstly, the relationship between the number of sides of a polygon and the number of triangles in a polygon and secondly, the

relationship between the number of triangles in a polygon and the sum of the interior angles of a polygon:

1. What is the relationship between the number of sides of a polygon and the number of triangles in a polygon?

2. What is the relationship between the number of triangles in a polygon and the sum of the interior angles of a polygon?
3. What is your conclusion about the sum of the interior angles of a polygon?

VI. CONCLUSION

The findings of the study from the analysis of the GSP activities in the first, second and third lessons indicate that the members of this LS group showed gradual improvement in their skills and confidence to design GSP activities for teaching the sum of the interior angles of a polygon after engaging in LS. In the first lesson, as shown in Figures 1a and 1b, they designed a GSP activity that allowed students to: (1) construct a circle using the Compass Tool; (2) construct points on the circle using the Point Tool; (3) label the points using the Text Tool; (4) construct the segments using the Straightedge Tool; (5) construct the polygon interiors using the Polygon Interior command in the Construct menu; (6) hide the circles using the Hide Circles command in the Display menu; (7) construct the diagonals using the Straightedge Tool; (8) measure the interior angles using the Angle command in the Measure menu; and (9) calculate the sum of the interior angles using the Calculate command in the Measure menu. In addition, as shown in Figure 1b, they were able to construct congruent circles using the Circle By Center+Point command in the Construct menu.

In the second lesson, as illustrated in Figure 2, the LS group members could simplify their GSP activity by allowing students to: (1) construct a triangle, quadrilateral, pentagon and hexagon using the Straightedge Tool in a single sketch and label the vertices of the polygons using the Text Tool; (2) measure all the interior angles of the polygons using the Angle command in the Measure menu; (3) calculate the sum of the

interior angles of the polygons using the Calculate command in the Measure menu; and (4) construct all the diagonals of the polygons using the Straightedge Tool.

Finally in the third lesson, as evidenced in Figure 3, the LS group members were able to revise the GSP activity by providing students the opportunities to (1) construct a heptagon, octagon, nonagon, and decagon and measure all the interior angles of the polygons; (2) calculate the sum of the interior angles of the polygons; and (3) construct all the diagonals of the polygons. Most importantly, the LS group members were able to design dynamic and interesting GSP activities that allow students to click and drag any vertex of the polygons and observe the relationship among the number of sides, number of triangles and the sum of the interior angles of the polygons. Furthermore, the students were provided with guided questions in the final GSP activity sheet to help them conclude that the sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$.

However, in this paper the researcher only managed to share the gradual improvement in the skills and confidence to design GSP activities of one selected LS group for teaching the sum of the interior angles of a polygon after engaging in LS. The researcher acknowledged the limitations of observing the positive changes in all the participants' skills and confidence to design GSP activities after engaging in LS. Nevertheless, the researcher was very much encouraged by the positive attitude and commitment of the participants in designing and re-designing the GSP activities several times as revealed in their GSP activity sheets in the first, second and third lessons as well as their numerous consultations with the researcher.

In conclusion, LS provided an alternative means of enhancing the participants' skills to design GSP activities for teaching mathematics which in turn enhanced their confidence in using GSP to teach mathematics

in general and the topic in particular at the secondary school level.

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